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Investment Strategy

RISK AND UTILITY IN OUTCOME-ORIENTED INVESTING

"In their theory, mathematicians evaluate money in proportion to its quantity while, in practice, people with common sense evaluate money in proportion to the utility they can obtain from it." –Daniel Bernoulli, 1793

As outcome-oriented investing has grown in popularity and importance, we increasingly find ourselves having to speak the language of economic utility to our clients, who may not be familiar with it. The concept of economic utility was first proposed by Daniel Bernoulli in the 18th century. The theory and practical application was later revived, expanded upon, and popularized in the 20th century. One practical application lies squarely in the field of outcome-oriented investing. This paper serves as a primer on economic utility functions and its application to outcome-oriented investing, also known as goals-based investing.

WHAT IS OUTCOME-ORIENTED (GOALS-BASED) INVESTING?

Outcome-oriented investing refers to investment strategies designed to achieve a specific goal or set of goals. Examples of such goals include:

- For an individual: paying for a college education (e.g., college savings portfolio)
- For a defined-contribution plan: sustaining a targeted standard of living in retirement for all participant cohorts, regardless of when they were born (e.g., target date portfolio)
- For an endowment or charitable foundation: accommodating a targeted spending budget (e.g., commitment-based investment strategy)
- For an insurance company or defined-benefit pension plan: funding a set of liabilities that represent future claims or benefit payments (e.g., liability-relative investment strategy)

This contrasts with traditional returns-based investing in which the goal is stated more simply as outperforming a market-based benchmark. Outcome-oriented investing is appealing because it gets to the heart of what an investor cares about most. Sure, it may be nice that an investment strategy consistently outperforms its benchmark from year to year, but this may matter little if it fails to deliver the amount needed to pay the bills.

In many outcome-oriented investment problems including those listed above, the outcome depends not only on investment performance, but also on the amount of money invested or withdrawn by the investor. Any combination of under-contributing, over-withdrawing, or poor investment performance may result in failing to achieve the desired outcome.

Furthermore, timing can be everything. To illustrate, assume an investor with a million dollar portfolio wishes to make annual end-of-year withdrawals of seventy-one thousand dollars for twenty years. This objective can be achieved if the portfolio earns a steady 3.6% per year. If returns vary from year to year, however, the investor could fall short of the withdrawal goal even if the average rate of return earned over the full horizon is significantly higher – say, double that of the required steady rate. Figure 1 demonstrates this by showing two return scenarios that each average 7.2% per year. In Scenario 1, a string of poor returns early on results in the portfolio going broke in year 19. In Scenario 2, which uses these same returns but in the opposite order, the goal is not only achieved but the portfolio actually ends with a higher balance than it began.

Note that the average annual return (arithmetic mean), compound annual growth rate (geometric mean), and standard deviation of return is the same in each case, yet the withdrawing investor may either increase her wealth or go broke depending entirely on the sequence in which the returns arrive. Clearly, the average rate of return says very little about whether the outcome will be favorable or disastrous. This issue gives rise to the notion of outcome-oriented investing as a much-needed alternative to traditional returns-based investing.

THE RISK OF UNCERTAIN OUTCOMES

The workhorse of outcome-oriented investing is typically a stochastic model that aims to quantify the probability of all possible outcomes. Outcomes that exceed the goals result in surplus wealth. Outcomes that fail to meet the goals are said to result in shortfall. Thus, the term shortfall risk refers to the risk of failing to meet the investor's goals.

FIGURE 1: **Sequence of Returns Risk**

		Scenario 1 Bad Start		Scenario 2 Good Start	
Year	**Planned Contribution (Withdrawal)**	**Return**	**Balance**	**Return**	**Balance**
0	$1,000,000		$1,000,000		$1,000,000
1	(71,000)	-12.0%	809,000	7.0%	999,000
2	(71,000)	-9.0	665,190	5.0	977,950
3	(71,000)	-5.0	560,931	14.0	1,043,863
4	(71,000)	10.0	546,024	15.0	1,129,442
5	(71,000)	12.0	540,546	9.0	1,160,092
6	(71,000)	7.0	507,385	8.0	1,181,900
7	(71,000)	15.0	512,492	17.0	1,311,823
8	(71,000)	-5.0	415,868	-1.0	1,227,704
9	(71,000)	15.0	407,248	16.0	1,353,137
10	(71,000)	17.0	405,480	9.0	1,403,919
11	(71,000)	9.0	370,973	17.0	1,571,586
12	(71,000)	16.0	359,329	15.0	1,736,324
13	(71,000)	-1.0	284,736	-5.0	1,578,507
14	(71,000)	17.0	262,141	15.0	1,744,283
15	(71,000)	8.0	212,112	7.0	1,795,383
16	(71,000)	9.0	160,202	12.0	1,939,829
17	(71,000)	15.0	113,232	10.0	2,062,812
18	(71,000)	14.0	58,085	-5.0	1,888,672
19	(71,000)	5.0	-	-9.0	1,647,691
20	(71,000)	7.0	-	-12.0	1,378,968
	Arithmetic Mean	**7.2**		**7.2**	
	Geometric Mean	**6.8**		**6.8**	
	Standard Deviation	**9.0**		**9.0**	

The above hypothetical examples are shown for illustrative purposes only and do not reflect an actual investment. Investment returns are not guaranteed, cannot be predicted, and will fluctuate. All investments are subject to risk, including the possible loss of the money invested.

A common technique for modeling these outcomes involves the use of Monte Carlo simulation for variables that are uncertain, but whose probability distribution can be estimated.[1] The simulation process involves the projection of numerous (perhaps tens of thousands) potential scenarios that could unfold over time for each variable such that the probability distribution of these scenarios reasonably reflects their uncertainty in the real world. The outcome along each scenario is then computed and compared to the investor's goals. Dividing the number of scenarios in which the outcome falls short of the goals by the total number of scenarios modeled results in the probability of shortfall, sometimes referred to as the probability of failure.

Beware that by itself, the probability of shortfall is an incomplete measure of risk because it says nothing about the potential magnitude of shortfalls that may occur.[2] Risk is a function of both components. Specifically, shortfall risk can be measured actuarially as a pricing function:

Total Shortfall Risk = Probability of Shortfalls x Magnitude of Shortfalls

Thus, an outcome with high probability and low magnitude may have equivalent risk as an outcome with low probability and high magnitude.

Keep in mind that the risk measure is meant to be purely objective. How an investor feels about taking on risk is something completely different...

THE UTILITY OF UNCERTAIN OUTCOMES

"The price of the item is dependent only on the thing itself and is equal for everyone...the utility, however, is dependent on the particular circumstances of the person making the estimate." –Daniel Bernoulli, 1793

For a given investment proposition and set of goals, one may think of total shortfall risk as the "price level" of risk. Being an objective measure, this price level does not depend on who takes the risk.[3]

However, the utility that the investment proposition yields will depend on the particular circumstances of the individual who is contemplating it. Thus, whereas the risk measure is purely objective, the utility measure is purely subjective.

The branch of economics devoted to this area of study is known as expected utility theory. Here, the qualifying term "expected" conveys that the outcomes being evaluated are uncertain. Think of utility as the level of satisfaction derived from a good or service—in this case, an outcome-oriented investment strategy. The level of satisfaction associated with all possible outcomes, whether they result in shortfall or surplus, is known as the investor's utility function.

Outcome-oriented utility functions reflect the manner in which an investor's willingness to take risk changes depending on their level of wealth (or, their funded status, which is the level of wealth relative to the amount required to meet the goals). Investors may exhibit an attitude of decreasing, constant, or increasing risk aversion. There is no right or wrong attitude—it is simply a matter of preference. See Figure 2 for examples.

FIGURE 2: **Risk Aversion Attitudes**

Type of Risk Aversion	Attitude Toward Shortfall	Attitude Toward Surplus
Decreasing risk aversion as funded status improves Increasing risk aversion as funding status deteriorates	The greater the shortfall, the less willing to take the chance of making the situation even worse.	The greater the cushion, the more willing to take risk.
Constant risk aversion	A shortfall has no influence on willingness to risk the wealth.	A surplus has no influence on willingness to risk the wealth.
Increasing risk aversion as funded status improves Decreasing risk aversion as funding status deteriorates	The greater the shortfall, the more willing to take risks because that is the only hope for reaching goals.	The greater the cushion, the less the desire to risk losing it.

Utility functions are very flexible. For example, an investor may exhibit decreasing risk aversion with respect to shortfall (small misses are minor, but large misses are a disaster), but constant or increasing risk aversion with respect to surplus (prefer to lock in success rather than risk it by reaching for more).[4]

IMPLICATIONS

The purpose of outcome-oriented investing is to deliver satisfying results for investors with specific goals in mind. Because outcomes are uncertain, this task requires an understanding of the investor's attitude toward the uncertainty of achieving the goals. Armed with 1) a set of investment goals, 2) a model for

[1] The estimate is typically expressed using parameters such as mean, standard deviation, skewness, and kurtosis. The correlation between uncertain variables is also incorporated into these models.

[2] How unfortunate that so much financial analysis and guidance seems to rely on probability measures alone without regard to the potential magnitude of failure. Mathematicians and economists have warned of the shortcomings of this approach for over three centuries, going back at least to 1654 in the work of Blaise Pascal and Pierre de Fermat.

[3] A corollary of the price level of risk is the amount it would cost to hedge it.

[4] Loss aversion, the tendency to prefer avoiding losses to acquiring equivalent gains, can explain this type of risk behavior. This idea is part of a related branch of economics known as prospect theory.

projecting outcomes and measuring shortfall risk, and 3) a utility function that reflects the investor's attitude toward shortfall and surplus, it is straightforward to measure the utility provided by any particular investment strategy.

In determining the utility of any particular investment strategy, potential outcomes that fall short of the goals are penalized according to the investor's utility function, while outcomes that exceed the goals are rewarded according to the investor's utility function. Evaluating all of the potential outcomes in total gives a measure of overall utility for the strategy. This overall utility measure allows different strategies to be ranked, and the strategy that offers the greatest utility to be identified. Thus, the use of utility functions in outcome-oriented investment strategies provides a means for producing potential solutions that are more bespoke to investor needs.

FURTHER READING

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Appendix

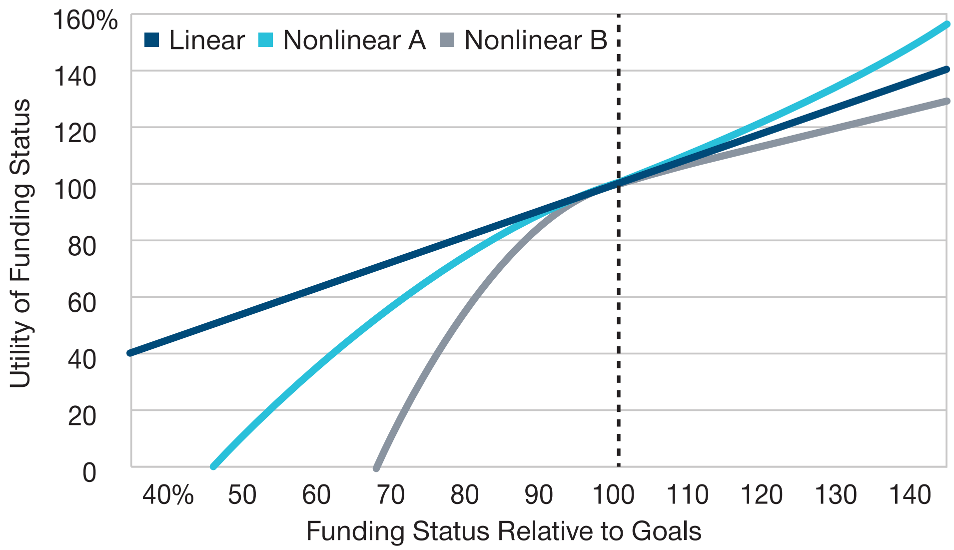
Since investment outcomes are uncertain, we are working not simply with utility, but expected utility. Expected utility can be computed on anything that is measurable: wealth, wealth relative to a liability, consumption, etc. The nature of the investor's goals will dictate what the utility function will measure. In general, however, we can say that utility functions should exhibit a preference for more wealth to less wealth, and more certainty to less certainty.[5]

The simplest form of utility function is linear, as in $U(w) = w$. This form, which exemplifies the constant risk aversion investor described in Figure 2, ranks wealth (or consumption or whatever) levels purely by expected values and is therefore said to be risk neutral. Investors with non-constant (decreasing or increasing) risk aversion can be described using nonlinear utility functions. Figure 3 illustrates with examples of a linear function and two nonlinear functions that apply different levels of risk aversion with regard to shortfall or surplus. Attitudes toward risk aversion can be seen in the degree of curvature in each line.

Consider the level of satisfaction with various outcomes for investors with these utility functions. The fact that each line slopes upward continually conveys that each investor always expresses a preference for achieving more to achieving less. Each of them will be satisfied if the goal is 100% funded. For the risk-neutral investor labeled Linear, utility is always equal to the funded status. Conversely, risk-averse investors express greater levels of dissatisfaction with shortfalls and thus are less willing to accept the possibility of bad outcomes. The investor labeled Nonlinear B is more averse to shortfalls than the investor labeled Nonlinear A, who is more averse to shortfalls than the risk-neutral investor labeled Linear.

Note also that the satisfaction felt by exceeding the investment goals is greatest for Nonlinear A—in fact, the utility of surplus for this investor is even greater than that of the risk-neutral investor. Thus, Nonlinear A will be increasingly willing to risk surpluses in the hope of gaining even more, while Nonlinear B will be less willing to risk surpluses out of a concern for falling back into shortfall.

FIGURE 3: **Example of Utility Functions**



[5] There are exceptions. For example, gamblers may be risk seeking rather than risk averse.

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